Temperature Rise in PCB Traces
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Background

I built my first "electronic" device over 40 years ago. (I was really young at the time!) Over the intervening years, there have been dramatic changes in technology. Some of these changes include the shift from designing circuits with components to designing systems with IC's, the shift from high voltage vacuum tube requirements (say 250 volts, or so) to (mostly) low voltage requirements, and the subsequent decline in the relative number of designs where high voltage and high current requirements are an issue.

In the 60's almost all designers had to worry about the current carrying capacity of PCB traces on at least some of their designs. Now, some designers can go through an entire career without having to address this issue at all. As I looked at this I began to understand why the significant investigations into PCB trace temperature-vs-current (T-C) relationships are mostly over 25 years old!

The current T-C bible for most of us is the set of charts in IPC-D-275. (IPC) (Footnote 1) Yet there is a nagging concern about them when we use them: Are they current? Are we sure where they came from and can they be trusted? Some people say they were generated with only three or four points and then "French Curves" were used to create smooth lines between the points. Others say they have been redrawn so many times by so many artists that they only somewhat resemble the original data. And you only have to look at the incongruous result from some of them that up to 125 ma of current can flow through a conductor with zero cross-sectional area! (You know, the curves really should go through the origin!)

Then I ran across another set of data in an old (1968) copy of "Design News" (DN) (Footnote 2). McHardy and Gandi recently reported on an analysis where they tried to test a theoretical, mathematical model on the IPC and the DN data (Footnote 3) with some limited success. That was when I decided to do the same thing using a different, more analytical (I believe) approach. This paper is a report of that analysis.

Defining the Model

We can think of a model as a representation of reality. In the context of this paper I will use an equation to "model" the relationship between current and the temperature of a trace. If the model is realistic, then when I substitute variables into the equation, the result will (within reason) reflect the actual result that would be obtained in the physical world. We can "test" a model by looking at actual results, and see if the model would give similar results under the same conditions.

It is intuitive that the flow of current through a trace (power) will cause the temperature of the trace to increase. The formula for power is $I^2 * R$, so the relationship is probably not simply linear. The resistance of a trace (per unit length) is a function of its cross-sectional area (width times thickness). So the relationship between temperature and current, therefore, is probably a non-linear function of current, trace width, and trace thickness. But the ability of a trace to "shed", or dissipate, heat is a function of its surface area, or width (per unit length). At the same time the current is heating the trace, the trace is cooling through the combined effects or radiation, convection and conduction through its surface. Therefore, the relative effect of width in the overall model is probably different than thickness.

A common model in thermodynamics for this type of situation is:

$$I = k * \Delta T^{b1} A^{b2}$$

Eq. 1

where: I = current in amps,
       ΔT = change in Temperature above ambient, in degrees C,
       A = cross sectional area in square mils, and
       k, $\beta 1$ and $\beta 2$ are constants.

Indeed, this is the starting point for McHardy and Gandi. Substituting Width*Thickness (W*Th) for area, we obtain a slightly more general model:

$$I = k * \Delta T^{b1} W^{b2} Th^{b3}$$

Eq. 2

So far, so good. But how do we determine those coefficients?

Developing the model

Least-squares-fit and multiple regression are techniques that can be used to estimate a set of constants in a situation like this. Assume we have a set of actual data for current, temperature change, and the width and thickness of traces. Least squares fit is a technique that will generate the constants and therefore give us an estimate for the equation (model) from that actual data. If we then use the estimated equation to calculate what the result would be, for any individual observation, and then look at the DIFFERENCE between the estimated value and the actual value, that DIFFERENCE is called an error term or a residual. The least squares fit is a technique that finds the set of coefficients that minimizes ("least") the variance of the error terms. (Footnote 4).

The difference between least-squares-fit and regression analysis is the set of assumptions we make about the error terms. Most importantly, we assume that they are randomly distributed. If this is true, then we can make statistically valid statements about probabilities related to the coefficients of a model and the resulting estimates from the model. If the randomness assumption is not valid, then we can still estimate the coefficients, but we cannot make any legitimate statistical inferences abut them. (This is not necessarily bad; useful predictive equations can often be obtained even when the randomness assumption is not met.)

To estimate the coefficients for Eq. 1 or Eq. 2, it is convenient first to convert them to linear form. We can do this using logarithms, as follows:

$$Ln(I) = Ln(k) + \beta 1 * Ln(\Delta T) + \beta 2 * Ln(A)$$

Eq. 3

$$Ln(I) = Ln(k) + \beta 1 * Ln(\Delta T) + \beta 2 * Ln(W) + \beta 3 * Ln(Th)$$

Eq. 4

Where Ln() is the natural logarithm (to the base e). We will use these forms of the model to find the best fit coefficients for the data.

Data

The IPC and DN sources have charts relating temperature change and current for various trace configurations. The DN data provides information allowing the independent evaluation of length and width for the traces under study. The IPC data appears to, but in fact it does not. The data really is tabulated by cross-sectional area for 4 trace thicknesses. I took approximately 300 data points from these charts, more or less randomly, as the source data for the analysis.

The first question is whether this approach can introduce its own error? Obviously my estimation of data points will result in some error. But IF this error is RANDOM, then it will introduce no bias into the estimate of the coefficients (constants), which is what this is all about. This additional random error will have a marginal effect on the statistical inferences we can make. But as we will see, there is enough error from other sources that my errors (if any) in estimating the raw data is pretty insignificant!

Analysis of DN Data

The DN data included charts for three trace thicknesses, 1 oz., 2 oz, and 5 oz. copper traces. When all DN data is used in a regression analysis, using Eq. 3, we get the following estimate for Eq. 3 (see Table 1 and Footnote 5):

$$Ln(I) = -3.23 + .45*Ln(\Delta T) + .69*Ln(A)$$

which leads to this estimate of Eq. 1:

$$I = .04 * \Delta T^{.45} A^{.69} \qquad \text{Eq. 5}$$

Looking at a graph of this result (Fig. 1), the fit is obviously not really good, so let's change the model, as discussed above, to use Width*Thickness instead of simply Area (Eq. 4) That results in the following estimate (see Table 2):

$$Ln(I) = -3.69 + .45*Ln(\Delta T) + .79*Ln(W) + .53*Ln(Th).$$

If the effects of Width and Thickness were equal, then the individual coefficients for Ln(W) and Ln(Th) would be equal and would be the same as the coefficient for Ln(A) above. That they are not is one indication that the form factor of the trace (not simply its cross-sectional area) is important.

This result leads to this estimate of Eq. 2:

$$I = .025 * \Delta T^{.45} W^{.79} Th^{.53} \qquad \text{Eq. 6}$$

Above it was mentioned that one desirable characteristic in a regression analysis is that the residuals (error terms) be randomly distributed. Figure 2 is a graph of the actual current (I) and the current (I') predicted from Eq. 6. The fit is better than that in Fig. 1. But when we look at the graph of the error terms (I - I', Fig 3, in order of trace thickness) the error terms are CLEARLY not random. It appears that the residuals for the 2 oz trace are significantly shifted from those for 1 oz. and 5 oz.

One way of adjusting for, and evaluating, the effects of this problem is through the introduction of what is called a "Dummy" variable. This is a variable whose value is 0 (zero) for all cases except where the trace thickness is 2 oz, where the value of the Dummy variable is 1 (one). Introducing a Dummy variable into Eq. 4 results in:
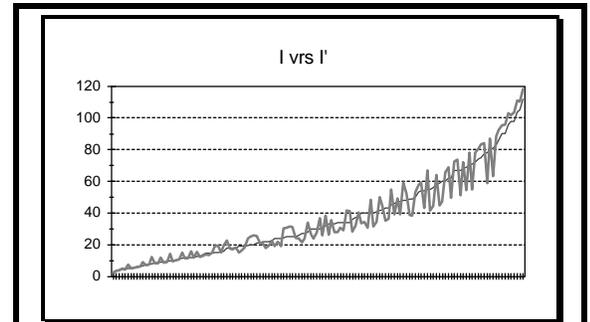
Figure 1
Graph of Actual vs Estimated Current from DN Data
Using Eq. 5
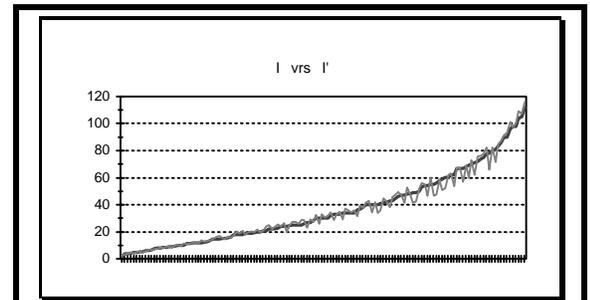
Figure 2
Improved Fit Using Eq. 6
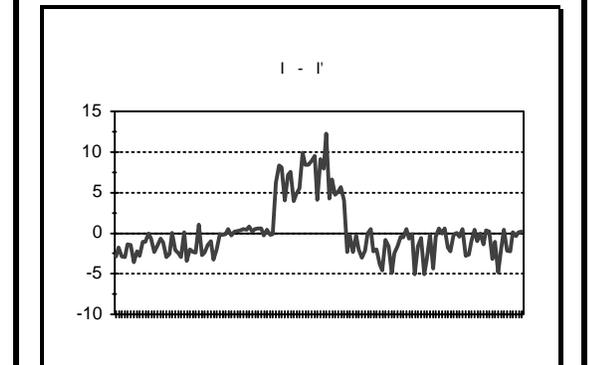
Figure 3
Graph of Residuals or Error Terms, Eq. 6

$$Ln(I) = Ln(k) + D + \beta1*Ln(\Delta T) + \beta2*Ln(W) + \beta3*Ln(Th) \qquad \text{Eq. 7}$$

Now, the result of the regression of this modified model is (see Table 3):

$$Ln(I) = -3.58 + .2*(D) + .46*Ln(\Delta T) + .76*Ln(W) + .54*Ln(Th). \qquad \text{Eq. 8}$$

Now D=0 for all but 2 oz. traces, so the result is really

$$Ln(I) = -3.58 + .46*Ln(\Delta T) + .76*Ln(W) + .54*Ln(Th) \qquad \text{for 1 and 5 oz. traces, and} \qquad \text{Eq. 9}$$
$$Ln(I) = -3.38 + .46*Ln(\Delta T) + .76*Ln(W) + .54*Ln(Th) \qquad \text{for 2 oz. traces.} \qquad \text{Eq. 10}$$

This results, in turn, in the following estimated models:

$$I = .028 * \Delta T^{.46} W^{.76} Th^{.54} \qquad \text{for 1 oz. and 5 oz. traces, and} \qquad \text{Eq. 11}$$
$$I = .034 * \Delta T^{.46} W^{.76} Th^{.54} \qquad \text{for 2 oz. traces} \qquad \text{Eq. 12}$$

The results of this model are graphed in Fig. 4(a). The residuals (error terms) are graphed in Figs. 4(b) (in Amps) and 4(c) (in percent.) In general, the model fits within 10% or 4 Amps, whichever is less.

(a)
Actual Current (I) &
Estimated Current (I')

(b)
Residuals (Amps)

(c)
Residuals (%)

Figure 4
Results of Eq. 8 (DN Data)
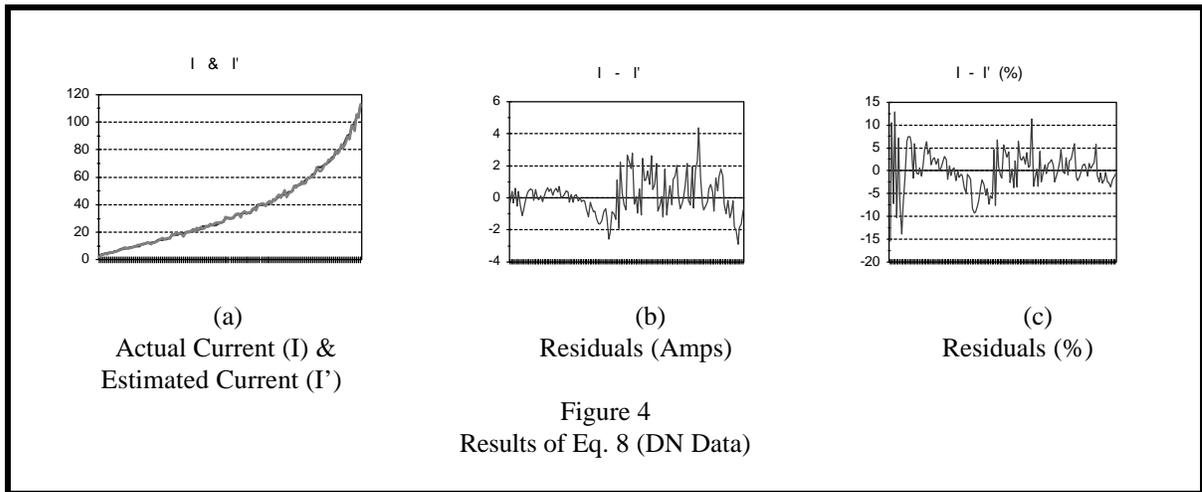
Summary of DN Data

The implications of this are quite interesting. Let's summarize the results so far:

$$I = .040 * \Delta T^{.45} A^{.69} \qquad \text{Adj. } R^2 = .961 \qquad \text{Eq. 5}$$
$$I = .025 * \Delta T^{.45} W^{.79} Th^{.53} \qquad \text{Adj. } R^2 = .990 \qquad \text{Eq. 6}$$
$$I = .028 * \Delta T^{.46} W^{.76} Th^{.54} \qquad \text{(for 1 oz. and 5 oz. traces, and)} \qquad \text{Adj. } R^2 = .997 \qquad \text{Eq. 11}$$
$$I = .034 * \Delta T^{.46} W^{.76} Th^{.54} \qquad \text{(for 2 oz. traces)} \qquad \text{Eq. 12}$$

The "Adjusted R^2" is a measure of "goodness of fit." In general, the higher the value of R^2, the better the model is at fitting the actual data. A "perfect" fit would result in an $R^2 = 1.0$, and a "perfect 'non-fit' " (which would be suspicious in itself!) would result in an R^2 of 0 (zero). No matter how we look at the data, the coefficient of the ΔT term is .45 or .46 . We can have a high confidence that this reflects a "true" relationship, at least for this data. Separating the cross-sectional area term (A) into its two components, width (W) and thickness (Th) results in significant improvement, and once that is done, those coefficients remain somewhat stable.

The impact of the Dummy variable is surprising. What it implies is --- all other things equal --- the 2 oz. traces can carry 21% more current than can the other traces! How can this be? I wasn't there to see the test conditions. The article summarizes the test conditions, but not nearly in enough detail to be able to evaluate what happened. But my personal opinion is that one or both of the following probably explain this result:

1. This kind of test is inherently difficult to set up and to control. The results reflect the normal variability that is to be expected from these kinds of investigations. However, if this were true, then the close consistency of the data for the 1 oz. and 5 oz. traces would not necessarily be expected.

2. Since all other results are so consistent, there was some variable that was not controlled as tightly as the researchers thought, and the results reflect conditions that were slightly different when the 2 oz. traces were fabricated and/or tested.

Analysis of IPC Data

The IPC data is graphed in IPC-D-275 for two conditions, external traces and internal traces. In a similar manner to the above, the IPC external data was used to fit the coefficients to the model in Eq. 3 with the following results (see Table 4 and Fig. 5):

$$I = .065 * \Delta T^{.43} A^{.68} \qquad\qquad\qquad \text{Eq. 13}$$

(a)	(b)	(c)
Actual Current (I) & Estimated Current (I')	Residuals (Amps)	Residuals (%)

Figure 5
Results of IPC External Data, Eq. 13
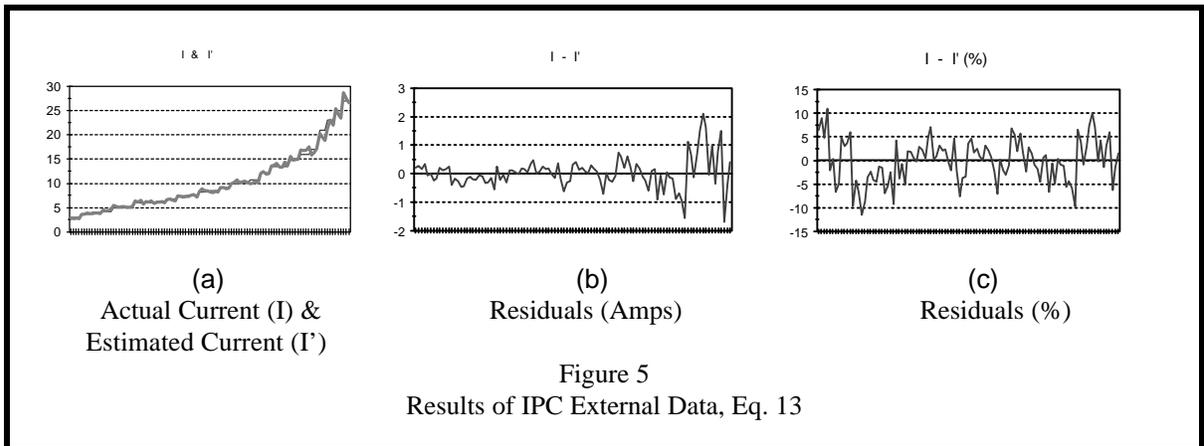
The IPC data does not provide a way of independently obtaining the width and thickness components of the cross-sectional data except by estimating them from the middle graph. When this is done, the results shown in Table 5 occur. Note that the coefficients for the width and thickness terms are (1) almost identical to each other, and (2) almost identical to the coefficient for the area term, above. All other results are virtually identical. This illustrates that there is no information to be gained from the IPC data from breaking down the IPC area numbers into their width and thickness components. This, therefore, implies that the IPC data was not taken with this idea in mind, or at least that it was ignored in the subsequent presentation of the data.

Comparing the results of this model for the two sets of data reveals:

$$I = .040 * \Delta T^{.45} A^{.69} \qquad \text{(DN data)} \qquad\qquad\qquad \text{Eq. 5}$$
$$I = .065 * \Delta T^{.43} A^{.68} \qquad \text{(IPC data)} \qquad\qquad\qquad \text{Eq. 13}$$

This data suggest that the fundamental model is the same for both sets of data (IPC and DN) but that all other things equal, the IPC implied currents are over 60% higher. This can be a little misleading, however. Consider this possibility: Use the DN model (Eq. 5) to calculate what the IPC currents would be for each observation of IPC ΔT and Area. Then compare this calculated IPC current (using the DN model, Eq. 5) to the actual IPC current from the chart. When this is done, we get the following relationship (see Table 6):

IPC Current (Amps) = .251+ 1.34*(Current Predicted From DN Eq. 5) $R^2 = .996$ Eq. 14

This shows that, on average, the IPC currents are shifted UP by 250 ma (remember the comment that they really should go through the origin?), and then are 34% higher than those implied by the DN model. But, then, DN's own 2 oz. trace data are higher than would be predicted by this model, also!

This time it is easier to accept that the reason is different test conditions. Although I have not been able to determine this precisely, I have reason to believe that the test conditions for the DN data collection and the IPC data collection were quite different (Footnote 6). I believe the IPC data were taken with the test board hung vertically, and the temperature change data were determined by the change in resistance of the trace under test. Since the temperature coefficient of resistivity is (supposedly) known for copper, then a change in resistivity can be directly correlated with a change in temperature. The DN data were taken with the test board hung horizontally, and the temperature change read with an infrared microscope. The data are remarkably close considering the fact that the data were taken (a) using different testing procedures on (b) different boards, (c) at different times, (d) by different people! It is especially remarkable that the coefficients for the primary variables are virtually identical.

IPC Internal Data

The IPC charts also include data for internal traces (the DN charts do not). Those data were fitted to Eq. 2 to compare the results with IPC's external data with the following results (see Table 7):

Ln(I) = -4.20 + .55*Ln(ΔT) + .74*Ln(A)

which leads to this estimate of Eq. 1:

$$I = .015 * \Delta T^{.55} A^{.74}$$ (IPC Internal)   Eq. 15

I have heard rumors (which I have not confirmed) that the IPC internal charts were simply derated 50% from the external ones. In a practical sense that is about the conclusion that could be drawn from, and is consistent with, the result of Eq. 15.

Conclusion

The relationship of current, change in temperature, and PCB trace cross-sectional area has been assumed to be of the form:

$$I = k * \Delta T^{b1} A^{b2}$$ Eq. 1

Analysis of two independent sets of data suggest this relationship is true, with the coefficients $\beta 1$ and $\beta 2$ being approximately .44 and .68, respectively. The Design News data suggest that this model can be improved by separating the area term into its components, Width * Thickness:

$$I = k * \Delta T^{b1} W^{b2} Th^{b3}$$ Eq. 2

When this occurs, the coefficient β1 does not significantly change, but β2 and β3 become .76 and ,54, respectively.

The constant term, k, however, varies considerably by data source and even within one set of data, suggesting that it is quite sensitive to variations in test conditions.

The IPC internal trace data suggest that currents be derated 50% (with respect to external traces) for the same degree of heating.

Temp Calculator

My company has created a freeware Windows calculator (PCBTEMP.EXE) for determining the relationship between current, trace configuration, and trace temperature rise. It is available for downloading from:
    www.UltraCAD.com
Follow the links to calculators.
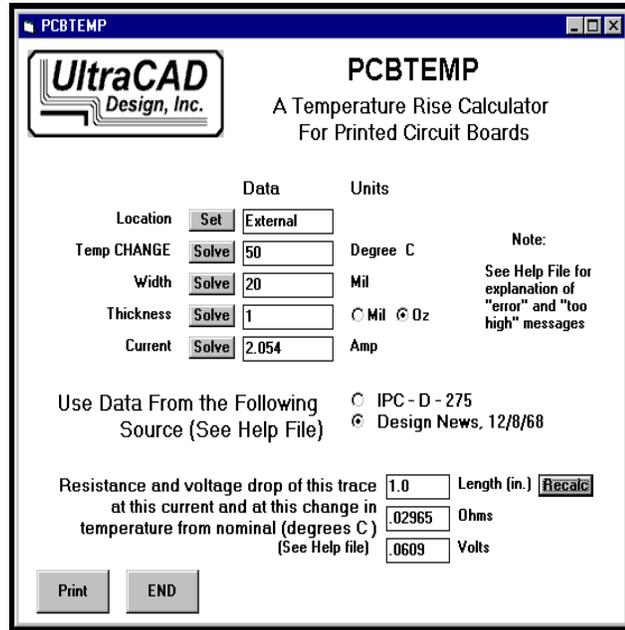
Figure 6
Example output from PCBTEMP.EXE

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Footnotes

1. ANSI/IPC-D-275, Design Standard for Rigid Printed Boards and Rigid Printed Board Assemblies, Figure 3-4, Page 10, IPC, September, 1991

2. "Printed Circuits and High Currents", Friar, Michael E. and McClurg, Roger H., Design News, Vol. 23, December 6, 1968, pp. 102 - 107.

3. "Empirical Equation for Sizing Copper PWB Traces," McHardy, John, and Gandhi, Mahendra, Presented at IPC Works '97, October 5-9, 1997, Arlington, VA

4. The formulas can be very complex, and any reasonable problem requires a computer to do the analysis. All major current spreadsheets can perform least squares fits and regression analyses. Almost any text for a first or second course in college level statistics will cover this topic.

5. The tables of coefficients and for ANOVA for the various results are included as an Appendix for those readers who understand them. A thorough understanding of these tables is not necessary to understand the fundamental conclusions that will be drawn from this analysis.

6. I am indebted to Ralph Hersey of Ralph Hersey and Associates, Livermore, CA., for insights into test procedures and the history of this kind of data. (Nevertheless, any errors and/or shortcomings in this analysis are purely my own.)

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Appendices

An Interesting Observation

(In the following analysis, "k" represents a constant, but not necessarily the *same* constant from step to step. This will keep the flow of the logic easier.)

Look at the form of Eq. 6:

$$I = k * \Delta T^{.45} W^{.79} Th^{.53}$$

Rearrange terms:

$$\Delta T^{.45} = k * I / W^{.79} Th^{.53}$$

Now, **approximately** square both sides:

$$\Delta T \approx k * I^2 / W^{1.5} Th$$

Recognize that Area (A) = W*Th

$$\Delta T \approx k * I^2 / A * \sqrt{W}$$

Further recognize that Resistance is proportional to 1/A, so

$$\Delta T \approx k * I^2 R / \sqrt{W}$$

This suggests that ΔT is directly proportional to power ($I^2 R$), which acts to heat the trace, and inversely proportional to the square root of W (surface area), which helps to cool the trace. Thus, the results of this analysis lead to a fairly reasonable, intuitive understanding of the dynamics involved.

ANOVA Tables Related to the Various Analyses

```
Multiple Regression Analysis
-----------------------------------------------------------------------------
Dependent variable: Ln_I
-----------------------------------------------------------------------------
                                   Standard           T
Parameter             Estimate       Error        Statistic        P-Value
-----------------------------------------------------------------------------
CONSTANT              -3.23158      0.12787        -25.2725         0.0000
Ln_DT                 0.450045      0.0237091       18.982          0.0000
Ln_Area               0.686565      0.0125318       54.7857         0.0000
-----------------------------------------------------------------------------

                      Analysis of Variance
-----------------------------------------------------------------------------
Source          Sum of Squares    Df   Mean Square    F-Ratio       P-Value
-----------------------------------------------------------------------------
Model               96.4864        2      48.2432      1646.46       0.0000
Residual            3.95567       135    0.0293012
-----------------------------------------------------------------------------
Total (Corr.)       100.442       137

R-squared = 96.0617 percent
R-squared (adjusted for d.f.) = 96.0034 percent
Standard Error of Est. = 0.171176
Mean absolute error = 0.142287
Durbin-Watson statistic = 0.215158
```

Table 1
Regression of Ln(I) vs Ln(DT) and Ln(A), DN data

```
Multiple Regression Analysis
-------------------------------------------------------------------------------
Dependent variable: Ln_I
-------------------------------------------------------------------------------
                                   Standard         T
Parameter            Estimate        Error      Statistic       P-Value
-------------------------------------------------------------------------------
CONSTANT             -3.68796      0.0681556     -54.1109        0.0000
Ln_DT                0.452776      0.0118993      38.0508        0.0000
Ln_W                 0.791638      0.0082062      96.4683        0.0000
Ln_Th                0.532173      0.00998182     53.3143        0.0000
-------------------------------------------------------------------------------

                         Analysis of Variance
-------------------------------------------------------------------------------
Source         Sum of Squares   Df  Mean Square   F-Ratio      P-Value
-------------------------------------------------------------------------------
Model               99.4532      3      33.1511    4492.18      0.0000
Residual          0.988884     134    0.00737973
-------------------------------------------------------------------------------
Total (Corr.)       100.442     137

R-squared = 99.0155 percent
R-squared (adjusted for d.f.) = 98.9934 percent
Standard Error of Est. = 0.0859054
Mean absolute error = 0.0672203
Durbin-Watson statistic = 0.487489
```

Table 2

Regression of Ln(I) vs Ln(DT), Ln(W), and Ln(Th), DN Data

```
Multiple Regression Analysis
--------------------------------------------------------------------------------
Dependent variable: Ln_I
--------------------------------------------------------------------------------
                                    Standard         T
Parameter            Estimate         Error      Statistic       P-Value
--------------------------------------------------------------------------------
CONSTANT             -3.57889       0.0375705     -95.258         0.0000
Ln_DT                0.457264       0.00647729     70.5949        0.0000
Ln_W                 0.762972       0.00474267    160.874         0.0000
Ln_Th                0.540707       0.00545039     99.2051        0.0000
D                    0.200244       0.0111956      17.886         0.0000
--------------------------------------------------------------------------------

                          Analysis of Variance
--------------------------------------------------------------------------------
Source         Sum of Squares    Df  Mean Square   F-Ratio      P-Value
--------------------------------------------------------------------------------
Model               100.152        4      25.0379  11467.34      0.0000
Residual          0.290394       133    0.00218341
--------------------------------------------------------------------------------
Total (Corr.)       100.442       137

R-squared = 99.7109 percent
R-squared (adjusted for d.f.) = 99.7022 percent
Standard Error of Est. = 0.046727
Mean absolute error = 0.0335754
Durbin-Watson statistic = 1.29037
```

Table 3

Effects of Introducing Dummy Variable, D, Into The Regression (Refer to Table 2)

```
Multiple Regression Analysis
-------------------------------------------------------------------------------
Dependent variable: Ln_I
-------------------------------------------------------------------------------
                                    Standard          T
Parameter            Estimate         Error       Statistic        P-Value
-------------------------------------------------------------------------------
CONSTANT             -2.73791       0.0392918      -69.6815         0.0000
Ln_DT                 0.428273      0.00617235      69.3858         0.0000
Ln_A                  0.67321       0.00642648     104.756          0.0000
-------------------------------------------------------------------------------

                          Analysis of Variance
-------------------------------------------------------------------------------
Source            Sum of Squares   Df  Mean Square    F-Ratio        P-Value
-------------------------------------------------------------------------------
Model                 35.0315       2     17.5158     7894.06         0.0000
Residual               0.226323   102      0.00221885
-------------------------------------------------------------------------------
Total (Corr.)         35.2579     104

R-squared = 99.3581 percent
R-squared (adjusted for d.f.) = 99.3455 percent
Standard Error of Est. = 0.0471047
Mean absolute error = 0.0373772
Durbin-Watson statistic = 1.01609
```

Table 4
Regression of Ln(I) vs Ln(DT) and Ln(A), IPC (External) Data

```
Multiple Regression Analysis
-------------------------------------------------------------------------------
Dependent variable: Ln_I
-------------------------------------------------------------------------------
                                    Standard          T
Parameter            Estimate         Error       Statistic        P-Value
-------------------------------------------------------------------------------
CONSTANT             -2.73817       0.0391329      -69.9712         0.0000
Ln_DT                 0.428273      0.00614715      69.6703         0.0000
Ln_W                  0.672272      0.00646809     103.937          0.0000
Ln_Th                 0.68235       0.00886406      76.9794         0.0000
-------------------------------------------------------------------------------

                          Analysis of Variance
-------------------------------------------------------------------------------
Source            Sum of Squares   Df  Mean Square    F-Ratio        P-Value
-------------------------------------------------------------------------------
Model                 35.0356       3     11.6785     5306.56         0.0000
Residual               0.222278   101      0.00220077
-------------------------------------------------------------------------------
Total (Corr.)         35.2579     104

R-squared = 99.3696 percent
R-squared (adjusted for d.f.) = 99.3508 percent
Standard Error of Est. = 0.0469124
Mean absolute error = 0.0369706
Durbin-Watson statistic = 1.04776
```

Table 5
IPC (External) Results Using A = W*Th.
(Note almost no new information is obtained.)

```
Regression Analysis - Linear model: Y = a + b*X
--------------------------------------------------------------------------------
Dependent variable: I
Independent variable: Col_16
--------------------------------------------------------------------------------
                            Standard          T
Parameter      Estimate      Error        Statistic        P-Value
--------------------------------------------------------------------------------
Intercept      0.251202     0.102736       2.44513          0.0162
Slope          1.34477      0.0117182      114.758          0.0000
--------------------------------------------------------------------------------


                        Analysis of Variance
--------------------------------------------------------------------------------
Source          Sum of Squares   Df  Mean Square   F-Ratio     P-Value
--------------------------------------------------------------------------------
Model               4008.3       1      4008.3    13169.51     0.0000
Residual            31.3493     103    0.304363
--------------------------------------------------------------------------------
Total (Corr.)       4039.65     104

Correlation Coefficient = 0.996112
R-squared = 99.224 percent
Standard Error of Est. = 0.551691
```

Table 6

IPC External Current as a Function of DN Current Estimated From Eq. 5

```
Multiple Regression Analysis
--------------------------------------------------------------------------------
Dependent variable: Ln_I
--------------------------------------------------------------------------------
                            Standard          T
Parameter          Estimate      Error      Statistic        P-Value
--------------------------------------------------------------------------------
CONSTANT           -4.19966    0.0766892    -54.7621          0.0000
Ln_DT              0.545301    0.018251      29.8779          0.0000
Ln_Area            0.735127    0.0105064     69.9695          0.0000
--------------------------------------------------------------------------------

                        Analysis of Variance
--------------------------------------------------------------------------------
Source          Sum of Squares   Df  Mean Square   F-Ratio     P-Value
--------------------------------------------------------------------------------
Model               35.6781       2     17.839     2894.21     0.0000
Residual            0.351331     57   0.0061637
--------------------------------------------------------------------------------
Total (Corr.)       36.0294      59

R-squared = 99.0249 percent
R-squared (adjusted for d.f.) = 98.9907 percent
Standard Error of Est. = 0.0785092
Mean absolute error = 0.0563861
Durbin-Watson statistic = 0.925714
```

Table 7

Regression of Ln(I) vs Ln(DT) and Ln(A), IPC (Internal) Data